RESPONDING TO YIELD UNCERTAINTY BY DIVERSIFICATION

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ABSTRACT

A single item inventory model with one retailer and two suppliers is considered. The retailer does not receive all the quantity ordered. It is assumed that a random function of the lot size is actually delivered by the suppliers. The cases of binomial yield and stochastically proportional yield are considered. The retailer responds to the yield uncertainty by ordering different amounts from the suppliers. In the model the demand rate is constant, the replenishment is instantaneous, and backordering is allowed. The paper emphasizes the nature of the optimal policy (in its class) and its implications. The conditions under which diversification between the suppliers is profitable are discussed with the help of some numerical findings.

KEY WORDS

Inventory Theory, Yield Uncertainty, Diversification, Supply Chain, Responsive Inventory Management

1. INTRODUCTION

In this paper, we consider an inventory model with one retailer and two suppliers. The suppliers sell the same item at different prices and they have different yield distributions. What is meant by a yield distribution is a stochastic function that maps the order quantity to a shipment quantity. For a given order quantity, the amount shipped by the supplier has a known distribution.

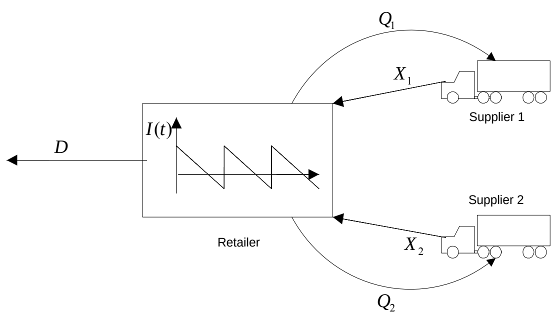

Figure 1. The inventory system under consideration

The retailer has to respond to this supply uncertainty and decide how much to order from each of these suppliers. The criterion for the retailer is his long-run average cost rate. His objective is to minimize his cost rate. The determination of the optimal order quantity from each supplier requires a steady-state analysis.

The retailer is employing a continuous review inventory policy. The demand for the item at the retailer, which drives the whole system, is occurring at a constant rate, D. It is assumed that when shortage occurs, the unfulfilled demand is fully backordered at a constant unit shortage cost per unit time. The unit holding cost for the inventory is the same for items obtained from the both suppliers. This makes sense since the items obtained from both suppliers are indistinguishable and are sold at the same retail price.

The retailer pays the suppliers for the quantity he orders, not the quantity he receives. Although it makes sense to apply the opposite in many cases, the results of the model can be readily converted with a minor modification. The yield distributions of the suppliers are assumed to be stationary and independent. Two types of yield distribution are considered and contrasted in this work: binomial yield and stochastically proportional yield. The replenishments are assumed to be instantaneous. The inventory policy class considered is (Q, r), which becomes for our model (Q_1, Q_2, r).

There is vast literature on the yield models in inventory control. These models are both for the EOQ setting and the periodic review setting. Our model is related with the ones with an EOQ setting. One of the leading papers in this setting is by Silver [1]. More recently, Mazzola, McCoy and Wagner [2] consider binomial yield, while Gerchak, Vickson, and Parlar [3] consider proportional yield. All these papers assume a single supplier unlike ours. For a thorough review of the extensive literature on yield models we relegate the reader to the paper by Yano and Lee [4].

Since our work incorporates two distinct suppliers, we would like to mention previous papers with such a setting. Anupindi and Akella [5] consider a periodic review inventory system with two suppliers. They investigate three different uncertainty scenarios in deliveries. One of these scenarios corresponds to stochastically proportional yield. The work by Parlar and Wang [6] published in 1993, is based on stochastically proportional yield and incorporates special cases of some the results presented in this paper. A recent interesting paper by Erdem and Ozekici [7] deals with diversification between two suppliers for a random capacity yield structure.

In this paper we extend the findings of Parlar and Wang [6] for the stochastically proportional yield and provide interesting results for the binomial yield. The findings for both cases are interestingly very different in nature. This takes us to the conclusion that diversification is beneficial only under certain kinds of uncertainty. In the next section, the mathematical model that constitutes the basis for all our findings is presented. Then follows a section on the generalization of the model to more than two suppliers. In the fourth section, we discuss some numerical results. Then, our paper concludes with the practical implications of our findings.

2. MATHEMATICAL MODEL

The decision parameters in our model are Q_1, Q_2, and i. While Q_1 and Q_2 represent the order quantities from supplier 1 and 2, respectively, i represents the reorder level for the inventory system. The random variables are X_1 and X_2, which are the quantities received from the suppliers. Of course these depend on the quantities ordered, Q_1, Q_2. But the mapping from Q_1, Q_2 to X_1, X_2 is stochastic. The total quantity received is represented by X ($X= X_1+X_2$). The relevant cost parameters are

- K: Fixed cost of ordering
- c_H: Unit holding cost per unit time
- c_S: Unit shortage cost per unit time
- c_1: Unit purchasing cost from supplier 1
- c_2: Unit purchasing cost from supplier 2.

We are considering in our work two types of yield structure: binomial yield and stochastically proportional yield. The binomial can be defined by the following distribution function

$$P(X_i = x / Q_i) = \binom{Q_i}{x} p_i^{\,x} (1-p_i)^{Q_i - x} \quad i = 1,2$$

where p_1 is the probability of producing a good unit for supplier 1 and p_2 is the probability of producing a good unit for supplier 2. This yield structure is suitable to model quality problems in the incoming orders. In this case, p would be the probability that a given item of the order dispatched is of acceptable quality, where X would be the total number of items of acceptable quality. For the binomial yield,

$$E[X] = p_1 Q_1 + p_2 Q_2 \quad \text{and}$$
$$Var[X] = p_1(1-p_1)Q_1 + p_2(1-p_2)Q_2 .$$

The stochastically proportional yield can be defined by the following equation.

$$P(x \le X_i \le x + \Delta x / Q_i) = P\left(\frac{x}{Q_i} \le u_i \le \frac{x+\Delta x}{Q_i}\right) \quad i = 1,2$$

where u_1 and u_2 are independent random variables modeling the random fractions for supplier 1 and 2 respectively. These random fractions have the means μ_1, μ_2 and the variances σ_1^2, σ_2^2, respectively. Consequently,

$$E[X] = \mu_1 Q_1 + \mu_2 Q_2 \quad \text{and}$$
$$Var[X] = \sigma_1^2 Q_1 + \sigma_2^2 Q_2 .$$

The inventory behavior of the system described is depicted in Figure 2.

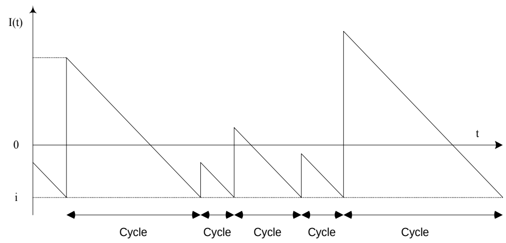

Figure 2. The inventory behavior at the retailer

The cycles are times between the arrivals of the orders. The control policy stipulates ordering of Q_1 and Q_2 from each supplier respectively every time the inventory level hits the level i. We can easily notice that each time of order arrival is a regeneration point for the process.

We have developed an approximate algorithm to find the optimal values of Q_1, Q_2, and i, minimizing the long-term average cost rate. Our algorithm works both for the binomial yield and stochastically proportional yield cases. We will not go into the details of our algorithm in this paper, but discuss the implications of our findings.

3. NUMERICAL RESULTS AND DISCUSSION

Our theoretical development stipulates the relation between model parameters and optimal values of decision variables that is summarized in Table 1.

Table 1. The effect of input variables on decision variables

K ↑	Q_1^* ↑	Q_2^* ↑	$\lvert i^* \rvert$ ↑	CR^* ↑
c_H ↑	Q_1^* ↓	Q_2^* ↓	$\lvert i^* \rvert$ ↑	CR^* ↑
c_S ↑	Q_1^* ↓	Q_2^* ↓	$\lvert i^* \rvert$ ↓	CR^* ↑

The effects of changing input parameters are visible in Table 2, where the optimal results are tabulated for a range of input parameters for the model with binomial yield. All the results supports the relations manifested in Table 1.

Table 2. Binomial yield case
($c_1 = 96$, $c_2 = 120$, $p_1 = 0.6$, $p_2 = 0.8$)

		$c_S = 50$				$c_S = 60$			
K	c_H	Q_1^*	Q_2^*	i^*	CR	Q_1^*	Q_2^*	i^*	CR
200	5	0.00	11.73	-0.85	193.140	0.00	11.64	-0.72	193.467
	10	0.00	8.66	-1.15	208.735	0.00	8.54	-0.98	209.554
	20	0.00	6.61	-1.51	227.589	0.00	6.45	-1.29	229.458
	30	0.00	5.77	-1.73	239.565	0.00	5.59	-1.49	242.420
	40	0.00	5.30	-1.89	248.216	0.00	5.10	-1.63	251.938
400	5	0.00	16.58	-1.21	210.802	0.00	16.46	-1.01	211.264
	10	0.00	12.25	-1.63	232.650	0.00	12.08	-1.38	233.808
	20	0.00	9.35	-2.14	258.904	0.00	9.13	-1.83	261.544
	30	0.00	8.16	-2.45	275.472	0.00	7.91	-2.11	279.487
	40	0.00	7.50	-2.67	287.311	0.00	7.22	-2.31	292.555
600	5	0.00	20.31	-1.48	224.355	0.00	20.16	-1.24	224.921
	10	0.00	15.00	-2.00	251.000	0.00	14.79	-1.69	252.419
	20	0.00	11.46	-2.62	282.931	0.00	11.18	-2.24	286.164
	30	0.00	10.00	-3.00	303.000	0.00	9.68	-2.58	307.918
	40	0.00	9.19	-3.27	317.295	0.00	8.84	-2.83	323.698

The results given in Table 3 are interesting because they demonstrate how the retailer responds to the changes in his environment by switching the source from which he orders. We see, for example, how the retailer switches from retailer 1 to retailer 2 as retailer 2 becomes more reliable.

Table 3. Binomial yield case
($K = 500$, $c_1 = 108$, $c_2 = 120$, $c_H = 20$, $c_S = 50$)

p_1	p_2	Q_1^*	Q_2^*	i^*	CR
0.60	0.60	13.944	0.000	-2.390	303.5209
	0.70	0.000	11.952	-2.390	293.9507
	0.75	0.000	11.155	-2.390	282.0227
	0.80	0.000	10.458	-2.390	271.5228
	0.90	0.000	9.296	-2.390	253.8562
0.70	0.60	11.952	0.000	-2.390	276.8078
	0.70	11.952	0.000	-2.390	276.8078
	0.75	11.952	0.000	-2.390	276.8078
	0.80	0.000	10.458	-2.390	271.5228
	0.90	0.000	9.296	-2.390	253.8562
0.75	0.60	11.155	0.000	-2.390	266.0227
	0.70	11.155	0.000	-2.390	266.0227
	0.75	11.155	0.000	-2.390	266.0227
	0.80	11.155	0.000	-2.390	266.0227
	0.90	0.000	9.296	-2.390	253.8562

This switching behavior is demonstrated more clearly in the Figure 3. In that figure, a subset of the input parameter space is partitioned into regions where different suppliers are preferable. As we can clearly observe, we are indifferent between suppliers over a line, which actually never happens in real life. Thus, ordering from the two suppliers simultaneously does not make sense when the yield structure is binomial.

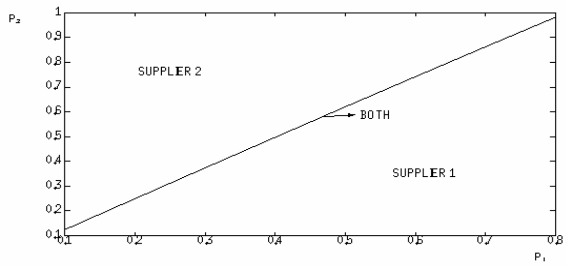

Figure 3. Optimal Choice of Supplier
($c_1 = 80$, $c_2 = 100$, $c_H = 20$, $D = 1$)

Similar behavior is observed in the stochastically proportional case, as it can be observed in Table 4. In the parameter range depicted in this table supplier 2 is always preferred. But there is also a range of parameters at which it is wise to order from both suppliers as in can be seen in Table 5. This is not something that is observed in the binomial yield case. This is due to variance reduction that is effective in the stochastically proportional case, when more than one supplier is used. This is similar to portfolio effect in finance that is used to reduce the risk. But this effect is not at work in the binomial yield case. This is due to the fact that the variance of the yield distribution is linear in binomial case, while it is quadratic in the stochastically proportional case. Thus, the variance reduction occurs as the order quantity is increased from a single supplier. Thus, there is no reason to diversify the orders using different suppliers.

Table 4. Stochastically proportional yield
($c_1 = 120$, $c_2 = 135$, $\mu_1 = 0.6$, $\mu_2 = 0.8$, $\sigma_1^2/\sigma_2^2 = 1.5$)

		$c_S = 50$				$c_S = 60$			
K	c_H	Q_1^*	Q_2^*	i^*	CR	Q_1^*	Q_2^*	i^*	CR
200	5	0.00	11.47	-0.83	212.356	0.00	11.38	-0.70	212.676
	10	0.00	8.45	-1.13	227.911	0.00	8.34	-0.95	228.710
	20	0.00	6.43	-1.47	246.514	0.00	6.28	-1.26	248.331
	30	0.00	5.59	-1.68	258.179	0.00	5.42	-1.45	260.938
	40	0.00	5.11	-1.82	266.464	0.00	4.93	-1.58	270.052
400	5	0.00	16.22	-1.18	230.419	0.00	16.10	-0.99	230.871
	10	0.00	11.95	-1.59	252.416	0.00	11.79	-1.35	253.547
	20	0.00	9.09	-2.08	278.725	0.00	8.89	-1.78	281.294
	30	0.00	7.91	-2.37	295.222	0.00	7.67	-2.05	299.124
	40	0.00	7.23	-2.57	306.938	0.00	6.98	-2.23	312.013
600	5	0.00	19.86	-1.44	244.278	0.00	19.72	-1.21	244.832
	10	0.00	14.64	-1.95	271.219	0.00	14.44	-1.65	272.604
	20	0.00	11.14	-2.55	303.441	0.00	10.88	-2.18	306.588
	30	0.00	9.68	-2.90	323.646	0.00	9.39	-2.50	328.425
	40	0.00	8.86	-3.15	337.995	0.00	8.55	-2.74	344.211

Table 4. Stochastically proportional yield
($c_1 = 90$, $c_2 = 120$, $\mu_1 = 0.6$, $\mu_2 = 0.8$, $\sigma_1^2/\sigma_2^2 = 1.5$)

		$c_S = 50$				$c_S = 60$			
K	c_H	Q_1^*	Q_2^*	i^*	CR	Q_1^*	Q_2^*	i^*	CR
200	5	4.19	8.39	-0.84	193.345	4.16	8.33	-0.70	193.666
	10	3.09	6.19	-1.13	208.775	3.05	6.10	-0.96	209.580
	20	2.36	4.71	-1.48	227.180	2.30	4.60	-1.27	229.009
	30	2.05	4.10	-1.69	238.676	1.99	3.98	-1.46	241.453
	40	1.88	3.76	-1.84	246.814	1.81	3.62	-1.59	250.421
400	5	5.93	11.86	-1.19	211.299	5.89	11.78	-1.00	211.754
	10	4.37	8.75	-1.60	233.121	4.32	8.63	-1.36	234.259
	20	3.33	6.66	-2.09	259.148	3.25	6.51	-1.79	261.736
	30	2.90	5.80	-2.39	275.407	2.81	5.62	-2.06	279.333
	40	2.66	5.31	-2.60	286.915	2.56	5.12	-2.25	292.017
600	5	7.27	14.53	-1.45	225.076	7.21	14.42	-1.22	225.633
	10	5.36	10.72	-1.96	251.802	5.29	10.57	-1.66	253.196
	20	4.08	8.16	-2.56	283.679	3.99	7.97	-2.19	286.848
	30	3.55	7.10	-2.93	303.592	3.44	6.89	-2.53	308.400
	40	3.25	6.51	-3.18	317.686	3.14	6.27	-2.76	323.935

Thus, the switching behavior in the stochastically proportional case is of a more complex nature. There is a region between single supplier regions, where it is profitable to use both suppliers in order to create a smaller variation in the yield. This is graphically manifested in Figure 4.

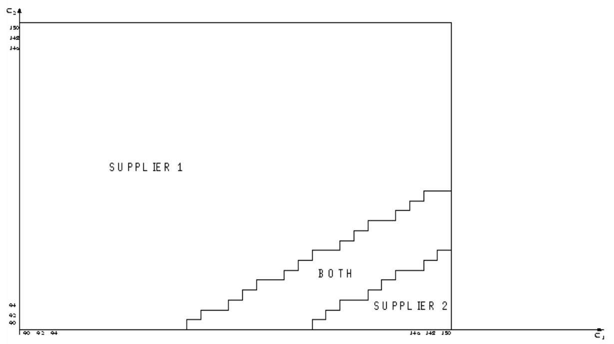

Figure 3. Optimal Choice of Supplier
($K = 500$, $c_H = 20$, $c_S = 50$, $\mu_1 = 0.8$, $\mu_2 = 0.6$, $\sigma_1^2 = 0.164$, $\sigma_2^2 = 0.148$)

4. SUMMARY AND CONCLUSION

In this paper, we report on the implications of a method we developed to find the optimal order quantities from two suppliers and the optimal reorder point for a continuous review inventory system. The discussion of the implications is based on a set of numerical findings that we present.

The determination of order quantities from the suppliers directly relates to the important problem of supplier selection in supply chain management. The suppliers are basically modeled with their order yield structures and unit prices. We see that the optimal strategy can be quite different depending on the setting. In the case of binomial yield, there is always a preferred supplier. This is what is actually advocated in the supply chain literature, although for different reasons. They usually claim that a close integration with a few trusted partners should bring tangible benefits to the enterprise. This makes a lot of sense from our paper's perspective, if it is to bring down the yield uncertainty.

But, we also observe that there may sometimes be a benefit to diversify the supply sources. This can be used to decrease the uncertainty by creating a portfolio effect on the variation. Thus, there is no single answer to the "existential" question, "To diversify or not to diversify?" It all depends on the setting considered.

REFERENCES

[1] Silver E. A., "Establishing the Reorder Quantity When the Amount Received is Uncertain," *INFOR*, 14(1), 1976, 32-39.

[2] Mazzola J. B., McCoy W. F., and Wagner H. M., "Algorithms and Heuristics for Variable Yield Lot Sizing," *Naval Research Logistics*, 34, 1987, 67-86.

[3] Gerchak Y., Vickson R. G., and Parlar M., "Periodic Review Production Models with Variable Yield and Uncertain Demand," *IIE Transactions*, 26, 1988, 19-24.

[4] Yano C. A. and Lee H. L., "Lot Sizing with Random Yields," *Operations Research*, 43, 1995, 311-334.

[5] Anupindi R. and Akella R., "Diversification Under Supply Uncertainty," Management Science, 39(8), 1993, 944-963.

[6] Parlar M. and Wang D., "Diversification Under Yield Randomness in Inventory Models", *European Journal of Operational Research,* 66, 1993, 52-64.

[7] Erdem A.and Ozekici S. " A Single Period Inventory Model with Two Vendors and Random Yield," Working Paper, Bogazici University, 1999.