A Model and Case Study for Efficient Shelf Usage and Assortment Analysis

M. Murat Fadıloğlu*

Oya Ekin Karaşan †

Mustafa Ç. Pınar‡

Department of Industrial Engineering

Bilkent University

06800 Ankara, Turkey.

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Abstract

In the rapidly changing environment of Fast Moving Consumer Goods sector where new product launches are frequent, retail channels need to reallocate their shelf spaces intelligently while keeping up their total profit margins, and to simultaneously avoid product pollution. In this paper we propose an optimization model which yields the optimal product mix on the shelf in terms of profitability, and thus helps the retailers to use their shelves more effectively. The model is applied to the shampoo product class at two regional supermarket chains. The results reveal not only a computationally viable model, but also substantial potential increases in the profitability after the reorganization of the product list.

Key words: retailing, assortment optimization, demand substitution.

1 Introduction

One of the critical issues in Fast Moving Consumer Goods (FMCG) sector is the rapid pace of the business where new retail products are launched frequently. Firms operating in this sector are forced to adopt operational strategies that would enable them to keep up with the fast changing market

*mmurat@bilkent.edu.tr, corresponding author

†karasan@bilkent.edu.tr

‡mustafap@bilkent.edu.tr

conditions. At the retail store level, the new products have to share common shelf space with the already existing products. The sheer increase in the number of different products or SKUs (SKU: stock keeping units as they are called in the sector) causes product pollution in stores. On the other hand, the firms supplying retail stores are looking for ways to increase their visibility through proliferating their SKUs to conquer shelf space. Both of these phenomena force retailers to eliminate certain SKUs from their shelves in favor of others. However, the criteria used in this elimination/replacement are not entirely clear.

This study was initiated after we were approached by the local office of an international FMCG conglomerate. They complained that the product pollution, which is especially acute in the small and medium size retailers, was hurting their sales. They stated that these retailers are not aware of the phenomenon and its repercussions on their own profitability. Since the retailers were not taking any measures about the unreasonable number of SKUs they are carrying on their shelf space, they were also not allowing the products of this FMCG conglomerate to realize their full potential on this retail channel. They believed that sales they were losing was more significant compared to other producers, since they were the market leaders in most product categories. The local office request was for us to develop a tool that the retailers could use to optimize their product mix on their shelves, which would also quantify the gain they can achieve by eliminating some SKUs. If the retailers implemented the recommendations of such a tool, this would reduce the product pollution and thereby, indirectly, benefit the sales of the conglomerate.

Against this background the purpose of the present paper is to develop a viable optimization model which would prevent product pollution and simultaneously achieve an efficient assortment by judicious partitioning of the shelf space among the variety of products. The model is applied to a specific hair care product class (shampoo) in two local chain stores and tested with actual data. The results summarized in the paper have been conveyed to the retail chains' managements and the resulting tool has been presented for their use.

Our contribution in this paper is an optimization model that improves the product mix on the shelf for any given product class at a retailer. There are many models in the literature with the same purpose. Yet, most models are complicated and have extensive data requirements. We claim that our model needs minimal data and can be used at/by almost any retailer that keeps track of their sales without complication. This model should be applicable to many smaller size retailers at which limited data is available.

The rest of this paper is organized as follows. In Section 2 we provide a brief overview of the related literature. Section 3 is devoted to the description and discussion of our model. A case study and results are reported in Section 4. Possible extensions and conclusions are given in Section 5.

2 Literature Review

The literature in the area of assortment management is quite large. We refer the readers to Mahajan and van Ryzin (1998) and Kök et al. (2005) for extensive reviews. Here we provide a brief review of the literature directly related to our manuscript.

An important work by Corstjen and Doyle (1981) provides an optimization model for shelf space allocation assuming a multiplicative self and cross space elasticity structure. They propose store experiments in order to estimate the parameters of their complicated elasticity structure. Since their estimation procedure is not practically viable, its applicability is limited. Inventory considerations are omitted in this model as is the case in ours. Urban (1998) extends it to cover inventory considerations as well.

A critical part of our model is the assumption that the customers who cannot find the SKU of their preference will substitute that SKU with another one. In our study, we assume that substitution occurs within product categories. A version (without the categories) of such substitution can be observed in inventory models incorporating demand substitution (see Netessine and Rudi (2003) and references therein). van Ryzin and Mahajan (1999) propose a newsboy model to determine the purchase quantities of different SKUs within a category. The paper makes use of a multinomial logit choice model to determine the individual purchase decisions. Each customer may choose to purchase among the listed products or may choose not to puchase anything. If the product of their choice in not available, the sale is lost. They show that the optimal assortment has a simple structure for their stylistic model in which the unit prices are identical. Majahan and van Ryzin (2001) also allow dynamic substitution to occur when the SKU demanded is not in stock and use sample path analysis to obtain structural results. Smith and Agrawal (2000) investigate the effect of substitution on the optimal base-stock levels in multi-item inventory systems. Their formulation leads to an integer program. Cachon et al. (2005) also incorporate customer search into the choice model which is similar to the one in van Ryzin and Mahajan (1999). There are also many papers in the literature that consider the determination of the optimal stocking levels given that assortment is already selected.

In a recent work, Kök and Fisher (2006) provide an assortment optimization model that takes into account the effect of shelf space allocation on customer choice as well as the effect of inventory policy on stockout based substitutions. Their model is quite elaborate and has extensive data requirements. In the paper, they also develop a procedure for the estimation of demand and substitution parameters and an iterative heuristic for the optimization model. They finally present the results of the application of their method, to a supermarket chain.

Our work is akin to the one of Kök and Fisher (2006), in the sense that we provide a tool that can be remotely applied in retailers and we present its application at two supermarket chains. However, our model is considerably simpler in terms of its structure. It does not take into account the elasticity of demand to shelf space nor the effects of the inventory policy. The supermarkets for which we devised our model do not have access to the kind of data necessary for the application of the method by Kök and Fisher (2006). We could only access monthly sales and price data. Our model provides a simple tool to exploit what is available in such settings.

3 Model

We group the products into categories according to their quality levels and prices. We assume K product categories are available. For convenience of presentation we define the category set $\mathcal{K} = \{1, \ldots, K\}$ and the SKU set within each category $i \in \mathcal{K}$, $\mathcal{N}_i = \{1, \ldots, N_i\}$.

We make the following important modeling assumption: when a SKU is taken off the shelf an estimated portion of the demand for that SKU is distributed to other SKUs. This portion is determined according to a "substitution ratio" s. The specific computation of this parameter for our target application is described in detail in Section 4.

We will assume that the following data from the retail stores are available for our model:

- The SKU list for on-shelf products at each category,
- Sales data for a predetermined time horizon for each SKU in each category,
- Profit margins of each SKU in each category,
- Probability (frequency) of buying another SKU from the same retailer when a given SKU is not carried in that retailer.

The cost of keeping one SKU on the shelf per period is referred to as c, and is assumed to be given. We discuss the meaning of this parameter and its estimation below.

Our model aims to find the optimum SKU list to be kept on shelf at retailers so as to maximize the total average profit per period. Therefore, the goal is to decide which SKUs to eliminate from the list to minimize product pollution on the shelves while maximizing total profit. Now, we describe the ingredients of the model.

Parameters:

- q_{ij}: average sales data per period for SKU j of category i,
- T_i: total sales per period in category i: $T_i = \sum_{j=1}^{N_i} q_{ij}$,
- p_{ij}: profit obtained when one unit of SKU j of category i is sold,

- c: cost per period of keeping one SKU on the shelf,
- s_i: substitution ratio for category i,
- d: minimum ratio for directly satisfied demand at each category, referred to as the *minimum conservation ratio.*

Decision Variables: Our decision variables are defined as follows:

- The decision to keep SKU i in category j is modeled using a binary variable:

$$x_{ij} = \begin{cases} 1 & \text{SKU } j \text{ of category } i \text{ is kept in the list} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

- U_i: total lost sales in category i
- $\tilde{q}_{ij}$: average estimated sales for SKU j of category i after substitution effect

Constraints: The constraints of our model, some of which are of a definitional nature, and therefore given only for convenience in the presentation, are defined as follows:

- Constraint to ensure that ratio of demand corresponding to undeleted SKUs to the total demand be no less than minimum conservation ratio d for each category i:

$$\sum_{j=1}^{N_i} q_{ij} x_{ij} \geq d\, T_i \text{ for all } i \in \mathcal{K}, \tag{2}$$

- Definition of total lost sales per period in category i due to unlisting:

$$U_i = T_i - \sum_{j=1}^{N_i} x_{ij} q_{ij} \text{ for all } i \in \mathcal{K}, \tag{3}$$

- Definition of average estimated sales per period for SKU j in category i after substitution:

$$\tilde{q}_{ij} = q_{ij} + \frac{s_i U_i q_{ij}}{T_i - U_i} \text{ for all } i \in \mathcal{K} \text{ and } j \in \mathcal{N}_i, \tag{4}$$

Our objective function consists of the difference of the total estimated profit after SKU elimination and the total cost of keeping the selected SKUs on the shelf:

$$Z \stackrel{def}{=} \sum_{i=1}^{K} \sum_{j=1}^{N_i} p_{ij} \tilde{q}_{ij} x_{ij} - c \sum_{i=1}^{K} \sum_{j=1}^{N_i} x_{ij} \tag{5}$$

Now, we are ready to define the optimization model for deciding the optimal SKU list that maximizes the total projected profit after substitution as choosing the values of the decision variables $x_{ij}, U_i, \tilde{q}_{ij}$ satisfying the constraints above and maximizing (5), i.e., as the model:

$$\max_{x_{ij}, U_i, \tilde{q}_{ij}} Z \text{ subject to } (1) - (2) - (3) - (4). \tag{6}$$

3.1 Discussion of the Model

Our first observation is that the optimization model (6) belongs to the class of nonlinear mixed integer programming problems. As such, it appears to be difficult to solve. However, as we shall see in Section 4, the model is solvable by off-the-shelf, state-of-the-art linear mixed-integer software packages for all sizes that are of practical interest and of relevance to our case study.

The basic idea behind the model is that when a SKU disappears from a retailer's shelves, the demand for that SKU is not completely lost. Some of the customers who cannot find the specific SKU they are looking for switch to other SKUs. This phenomenon is known as "product substitution". Thus, although some profit is forgone when a SKU is "de-listed", there is also some new profit for other SKUs due to demand that switches. We assume that product substitution occurs within the SKUs in the same category. This means that a customer that would like to buy a specific premium shampoo, would only be willing to substitute it with another premium shampoo. The proportion of demand that stays in the retailer for category i and switches to other products is a parameter of our model, s_i. This ratio is category dependent, since the substitution is more likely for low-end products, and less likely for premium products where brand loyalty is higher. Obviously, as s_i increases, it is easier to unlist certain category i items from the list since more of the demand is conserved and channeled to other SKUs.

An additional point which relates to product substitution needs to be clarified in order to formulate a model. One needs to quantify how the substituting demand will be distributed among the products of the same category that are kept in the list. The assumption our model espouses is that this distribution will be according to the relative weight of sales for each SKU with respect to total sales corresponding to all SKUs that are not deleted from the list. This distribution principle is embedded into the model with the constraint (4) that is used to define the variables $\tilde{q}_{ij}$, the estimated sales after substitution takes place. This constraint can be re-written as

$$\tilde{q}_{ij} = q_{ij} + s_i U_i \frac{q_{ij}}{T_i - U_i} \text{ for all } i \in \mathcal{K} \text{ and } j \in \mathcal{N}_i, \tag{7}$$

where the ratio $q_{ij}/(T_i - U_i)$ represents the fraction of the sales of the SKU j in category i with respect to the total sales in category i after de-listing. Multiplying this fraction with $s_i U_i$, the total substituting demand, one obtains the estimated additional sales due to substitution for the SKU j in category i.

It is reasonable to assume that the substitution ratios would not be dependent on the total quantity that is being substituted, since substitution is made by individual customers. When the proportion of the de-listed demand is not larger than 20% or 30%, one does not have much reason to question this assumption. Yet, if this proportion is for example 90%, it means that 90% of the demand would substitute to SKU's whose original demand is 10% of the total. Under such circumstances, the customer base would probably not be happy with the product mix presented on the retailer's shelves and the actual substution ratio would be lower. Moreover, since such a policy of satisfying only a small portion of customers' original demand would be detrimental for the image of the retailer, it is reasonable to assume that the model should have a bound on the ratio of the substituting demand. The minimum conservation ratio in our model represents this bound on the ratio of substituting demand for each category, as expressed in (2). The constraint in (2) ensures that the model does not suggest solutions in the region where modeling assumptions begin to break.

Another critical component of our model is the cost of keeping SKUs on shelf, which is linear on the number of SKUs on shelf. In the small and medium size retailers where our study was concentrated, we observed that there were quite a few SKUs with very small sale volumes. With such small sale volumes, they should not be worth the effort of organizing them on the shelf, keeping track of their inventories etc. Moreover, we realized that if such SKUs could be eliminated there would be drastic reduction in terms of the product pollution. Thus, c models all these indirect costs that are due to keeping one SKU on the list. Since this cost cannot be directly estimated by accounting methods, it has to be determined by asking the retail managers appropriate questions. It is obvious that as c increases, more elimination would take place and the optimal list would shrink.

Since most of the model constraints are of definitional nature, it is possible to collapse the constraints (3) and (4) into the objective function and obtain a more concise model. This concise form is instrumental in proving the results provided in the following sections, and it is presented below:

$$\max Z \stackrel{def}{=} \sum_{i=1}^{K} \sum_{j=1}^{N_i} \left(\left(p_{ij} q_{ij} (1 - s_i) - c\right) x_{ij} + s_i \left(\sum_{j=1}^{N_i} q_{ij} \right) \frac{\sum_j p_{ij} q_{ij} x_{ij}}{\sum_j q_{ij} x_{ij}} \right) \tag{8}$$

$$\text{subject to} \sum_{j=1}^{N_i} q_{ij} x_{ij} \geq d\, T_i \text{ for all } i \in \mathcal{K},\ x_{ij} \in \{0,1\} \text{ for all } i \in \mathcal{K}, j \in \mathcal{N}_i. \tag{9}$$

As can be seen in (8), the cost components that correspond to each category are simply added up to constitute the objective function. Moreover, the constraints of the optimization model given in (9) only involve the decision variables of a single category at a time. Thus, our model can be decomposed into K independent problems for each $i = 1, \ldots, K$:

$$\max Z_i \stackrel{def}{=} \sum_{j=1}^{N_i} \left(\left(p_{ij} q_{ij} (1 - s_i) - c\right) x_{ij} + s_i \left(\sum_{j=1}^{N_i} q_{ij} \right) \frac{\sum_j p_{ij} q_{ij} x_{ij}}{\sum_j q_{ij} x_{ij}} \right) \tag{10}$$

$$\text{subject to } \sum_{j=1}^{N_i} q_{ij} x_{ij} \geq d\, T_i,\ x_{ij} \in \{0,1\} \text{ for all } j \in \mathcal{N}_i. \tag{11}$$

We note that the original model (6) and the collapsed and decomposed model (10)-(11) are both non-convex and non-concave (neither quasi-convex nor quasi-concave) maximization problems over a set of integers. As such, they look quite difficult for numerical processing. Interestingly, a continuous relaxation that is immediately obtained by relaxing the integer (binary) requirement on the variables x_{ij} does not yield a tractable problem either, in the sense that the resulting maximization problem is not guaranteed to have neither quasi-convex, nor quasi-concave objective function. To see this, let us observe that the decomposed problem (10)-(11) for category i can be re-written in vector-matrix notation as

$$\max Z_i = f^T x + \frac{h^T x}{q^T x} \tag{12}$$

$$\text{subject to } q^T x \geq d\, T_i,\ 0 \leq x \leq e. \tag{13}$$

where $f \in \mathbb{R}^{N_i}$ with component j equal to $q_{ij} p_{ij}(1 - s_i) - c$, $h \in \mathbb{R}^{N_i}$ with component j equal to $s_i T_i p_{ij} q_{ij}$ and $q \in \mathbb{R}^{N_i}$ with components q_{ij}, while $x \in \mathbb{R}^{N_i}$ denotes the vector with components x_{ij}, for all $j \in \mathcal{N}_i$. We note immediately that the continuous relaxation model (12)-(13) would be equivalent to a linear programming problem were it not for the presence of the term $f^T x$ in the objective function. Without that term, the problem would fall into the realm of linear-fractional maximization with a quasi-convex and quasi-concave (hence, quasi-linear) objective function over a polyhedral set. It is well-known that such problems are easily transformed to equivalent linear programs. However, the presence of the term $f^T x$ renders the objective function neither quasi-convex, nor quasi-concave in general (see [8] for conditions regarding quasi-convexity or quasi-concavity of such functions). To see this, let us consider the level sets of the objective function

$$S_\alpha = \{x \in \mathbb{R}^{N_i} : f^T x + \frac{h^T x}{q^T x} \leq \alpha\}$$

for some scalar $\alpha \in \mathbb{R}$. Notice that the positivity of the denominator is guaranteed from the constraint $q^T x \geq d\, T_i$. Therefore, an equivalent expression for S_α is given by $\{x \in \mathbb{R}^{N_i} : (q^T x)(f^T x) + h^T x - \alpha q^T x \leq 0\}$. Equivalently, we have

$$S_\alpha = \{x \in \mathbb{R}^{N_i} : x^T \left(\frac{f q^T + q f^T}{2} \right) x + h^T x - \alpha q^T x \leq 0\}.$$

By definition of quasi-convexity, the set S_α should be a convex set. However, for this to hold true, one needs to guarantee positive semi-definiteness of the rank-2 matrix $\frac{fq^T + qf^T}{2}$, which is not true in general. A similar discussion with the sub-level sets $\{x \in \mathbb{R}^{N_i} : f^T x + \frac{h^T x}{q^T x} \geq \alpha\}$ encounters the same conclusion.

Therefore, even the continuous relaxation of our model does not seem to be, at least from a theoretical point of view, an easy continuous optimization problem. Nonetheless, in the sequel, we shall reveal several interesting features of the nonlinear model along with a careful linearization that contribute to its numerical solution in reasonable running times. These features include a property of the local optima of the problem, bounds for preprocessing, the special case of uniform profit, and finally an effective linearization scheme.

3.2 Local Optima

Consider the continuous relaxation of the collapsed and decomposed model (10)-(11) where the constraint bounding the ratio of substituting demand for each category is eliminated. In particular, consider the following continuous optimization problem:

$$(RP_i) \quad \max \sum_{j=1}^{N_i} \left(\left(p_{ij} q_{ij}(1-s_i) - c\right) x_{ij} + s_i \left(\sum_{j=1}^{N_i} q_{ij} \right) \frac{\sum_j p_{ij} q_{ij} x_{ij}}{\sum_j q_{ij} x_{ij}} \right) \tag{14}$$

$$\text{subject to } 0 \le x_{ij} \le 1 \text{ for all } j \in \mathcal{N}_i. \tag{15}$$

Theorem 1 *If $p_{ij} q_{ij}(1-s_i) - c > 0 \ \forall j$, then any local optimal solution of RP_i is integral.*

Proof: Let $x_i^* = (x_{i1}^*, x_{i2}^*, \ldots, x_{iN_i}^*)$ be a local maximum of RP_i. Without loss of generality assume that $0 < x_{il} < 1$. We shall proceed to show that under the condition of this theorem, x_i^* cannot be simultaneously as good as $x_i^* + \epsilon e_l^T$ and $x_i^* - \epsilon e_l^T$ where e_l is the unit vector of dimension N_i whose l^{th} component is 1, and ϵ is a sufficiently small positive number that keeps these two vectors feasible. Assume to the contrary that x_i^* is at least as good as both $x_i^* + \epsilon e_l^T$ and $x_i^* - \epsilon e_l^T$ in terms of objective value. In other words, we have

$$\sum_j p_{ij} q_{ij} x_{ij}^* \left(1 + s_i \left(\frac{\sum_j q_{ij}}{\sum_j q_{ij} x_{ij}^*} - 1 \right) \right) - c \sum_j x_{ij}^* \ge$$

$$\left(\sum_j p_{ij} q_{ij} x_{ij}^* + p_{il} q_{il} \epsilon \right) \left(1 + s_i \left(\frac{\sum_j q_{ij}}{\sum_j q_{ij} x_{ij}^* + q_{il}\epsilon} - 1 \right) \right) - c(\sum_j x_{ij}^* + \epsilon), \tag{16}$$

and similarly,

$$\sum_j p_{ij} q_{ij} x_{ij}^* \left(1 + s_i \left(\frac{\sum_j q_{ij}}{\sum_j q_{ij} x_{ij}^*} - 1 \right) \right) - c \sum_j x_{ij}^* \ge$$

$$\left(\sum_j p_{ij} q_{ij} x_{ij}^* - p_{il} q_{il} \epsilon \right) \left(1 + s_i \left(\frac{\sum_j q_{ij}}{\sum_j q_{ij} x_{ij}^* - q_{il}\epsilon} - 1 \right) \right) - c(\sum_j x_{ij}^* - \epsilon). \tag{17}$$

After some manipulation, (16) is equivalent to:

$$s_i \sum_j p_{ij} q_{ij} x_{ij}^* \left(\frac{q_{il} \sum_j q_{ij}}{\sum_j q_{ij} x_{ij}^* (\sum_j q_{ij} x_{ij}^* + q_{il}\epsilon)} \right) \ge p_{il} q_{il} + p_{il} q_{il} s_i \left(\frac{\sum_j q_{ij}(1 - x_{ij}^*) - q_{il}\epsilon}{\sum_j q_{ij} x_{ij}^* + q_{il}\epsilon} \right) - c. \tag{18}$$

Similarly, (17) is equivalent to:

$$s_i \sum_j p_{ij} q_{ij} x_{ij}^* \left(\frac{-q_{il} \sum_j q_{ij}}{\sum_j q_{ij} x_{ij}^* (\sum_j q_{ij} x_{ij}^* - q_{il}\epsilon)} \right) \ge -p_{il} q_{il} - p_{il} q_{il} s_i \left(\frac{\sum_j q_{ij}(1 - x_{ij}^*) + q_{il}\epsilon}{\sum_j q_{ij} x_{ij}^* - q_{il}\epsilon} \right) + c. \tag{19}$$

Now, from (18) and (19) we have:

$$\left(\sum_j q_{ij}x^*_{ij}(\sum_j q_{ij}x^*_{ij}+q_{il}\epsilon)\right)\left(p_{il}q_{il}+p_{il}q_{il}s_i\left(\frac{\sum_j q_{ij}(1-x^*_{ij})-q_{il}\epsilon}{\sum_j q_{ij}x^*_{ij}+q_{il}\epsilon}\right)-c\right)\leq q_{il}s_i\sum_j p_{ij}q_{ij}x^*_{ij}\sum_j q_{ij}\leq$$

$$\left(\sum_j q_{ij}x^*_{ij}(\sum_j q_{ij}x^*_{ij}-q_{il}\epsilon)\right)\left(p_{il}q_{il}+p_{il}q_{il}s_i\left(\frac{\sum_j q_{ij}(1-x^*_{ij})+q_{il}\epsilon}{\sum_j q_{ij}x^*_{ij}-q_{il}\epsilon}\right)-c\right) \tag{20}$$

which is, after some simplifications, equivalent to

$$p_{il}q_{il}(1-s_i)-c\leq 0,$$

leading to a contradiction. ∎

Interestingly the quantity $p_{ij}q_{ij}(1-s_i)-c$ that decides the integrality of local optima in the relaxed problem also appears in the discussion of the uniform profit case (Section 3.4). This quantity is instrumental in solving the problem in the uniform profit case. While it is hard to give a precise economic interpretation to this quantity, we can view it as a modified profit coefficient.

3.3 Bounds for Pre-processing

Now, we investigate lower and upper bounds on the change in the objective function value when we include a SKU in the list of SKUs on the shelf, while keeping everything else fixed. In other words, we assume to have a feasible vector x and concentrate on the item i,ℓ. We investigate the effect $\Delta Z_{i,\ell}$ on the objective function of making $x_{i,\ell}$ equal to one. Throughout this section, we use the collapsed and decomposed model (10)-(11).

Lemma 1

$$\Delta Z_{i\ell} = p_{i\ell}q_{i\ell}\left(1+s_i\left(\frac{\sum_j q_{ij}}{\sum_{j\neq\ell}q_{ij}x_{ij}+q_{i\ell}}-1\right)\right)-c-q_{i\ell}s_i\left(\frac{(\sum_j q_{ij})(\sum_{j\neq\ell}p_{ij}q_{ij}x_{ij})}{(\sum_{j\neq\ell}q_{ij}x_{ij}+q_{i\ell})(\sum_{j\neq\ell}q_{ij}x_{ij})}\right).$$

Proof: After writing out the difference between the objective function values corresponding to the cases where $x_{i,\ell}=0$ and $x_{i,\ell}=1$, respectively, we obtain

$$\begin{aligned}\Delta Z_{i\ell} &= p_{i\ell}q_{i\ell}\left(1+s_i\left(\frac{\sum_j q_{ij}}{\sum_{j\neq\ell}q_{ij}x_{ij}+q_{i\ell}}-1\right)\right)-c+\\ &\quad s_i\left(\sum_j q_{ij}\right)\left(\sum_{j\neq\ell}p_{ij}q_{ij}x_{ij}\right)\left(\frac{1}{\sum_{j\neq\ell}q_{ij}x_{ij}+q_{i\ell}}-\frac{1}{\sum_{j\neq\ell}q_{ij}x_{ij}}\right).\end{aligned}$$

After some manipulation, Lemma 1 is obtained. ∎

Proposition 1

$$p_{i\ell}q_{i\ell} - c - q_{i\ell}\frac{s_i}{d}(\max_j p_{ij}) \le \Delta Z_{i\ell} \le p_{i\ell}q_{i\ell}\left(1 + s_i\frac{1-d}{d}\right) - c - q_{i\ell}s_i(\min_j p_{ij}).$$

Proof: From Constraint 2 we know that

$$\sum_j q_{ij} \ge \sum_j q_{ij}x_{ij} \ge d\sum_j q_{ij}.$$

Therefore, we can give the following upper bound on $\Delta Z_{i\ell}$:

$$\Delta Z_{i\ell} \le p_{i\ell}q_{i\ell}\left(1 + s_i\left(\frac{\sum_j q_{ij}}{d\sum_j q_{ij}} - 1\right)\right) - c - q_{i\ell}s_i\frac{(\sum_j q_{ij})(\sum_{j\ne\ell} p_{ij}q_{ij}x_{ij})}{(\sum_j q_{ij})(\sum_{j\ne\ell} q_{ij}x_{ij})}.$$

Since $\sum_{j\ne\ell} p_{ij}q_{ij}x_{ij} \ge (\min_j p_{ij})\sum_{j\ne\ell} q_{ij}x_{ij}$, we obtain:

$$\Delta Z_{i\ell} \le p_{i\ell}q_{i\ell}\left(1 + s_i\frac{1-d}{d}\right) - c - q_{i\ell}s_i(\min_j p_{ij}).$$

Similarly, we can establish the lower bound

$$\Delta Z_{i\ell} \ge p_{i\ell}q_{i\ell} - c - q_{i\ell}\frac{s_i}{d}(\max_j p_{ij}).$$

∎

One can observe that the values of certain variables in the optimal solution can be determined by just checking the problem parameters based on the lower bound presented in the following proposition. This gives rise to the rule presented in Proposition 2. This rule is used for preprocessing the optimization model in order to reduce the problem size.

Proposition 2 *If the lower bound on $\Delta z_{i\ell}$ is positive or zero, i.e.,*

$$p_{i\ell}q_{i\ell} - c - q_{i\ell}\frac{s_i}{d}(\max_j p_{ij}) \ge 0$$

then SKU ℓ of category i is guaranteed to be in an optimal list, i.e., we can set $x_{i\ell} = 1$ without loss of optimality.

This proposition stipulates that the SKU ℓ in category i should be kept in the list, if the corresponding lower bound is greater or equal to zero. This is due to fact if this SKU is added to the list the objective function will improve by at least the value of the lower bound, irrespective of the composition of the list. Since adding an additional SKU to the list cannot violate any constraint of the program, we are never worse off when that SKU is in the list.

One can think of a similar proposition using the upper bound which would stipulate that a SKU should be kept off the list if the corresponding upper bound is nonpositive. Yet, the problem is that by keeping a SKU out of the list one can violate constraint (11). Thus, such an elimination can only be done if this constraint is nonbinding at the optimal point. Since this requires post-optimization processing, its use in pre-processing would be complicated. Still if constraint (11) is superfluous, i.e., $d = 0$, then the upper bound would also yield a pre-processing rule.

3.4 A Special Case: Uniform Profit

In this section, we present how the model simplifies when the profit margins are equal for each SKU within a category, i.e. when $p_{ij} = p_i$ for all $j = 1, \ldots, N_i$. Under this condition the objective function of the decomposed problem given in (10), would simplify to

$$Z_i = \sum_j (p_i(1 - s_i)q_{ij} - c)x_{ij} + p_i s_i \sum_j q_{ij} \tag{21}$$

If it were not for the constraint (11), in this setting the optimization problem would decompose to N_i independent subproblems involving one SKU at a time. This gives rise to the next proposition where x^* denotes an optimal solution and x^c denotes a candidate solution. Let $\Delta^u_{ij} = p_i(1-s_i)q_{ij} - c$.

Proposition 3

1. *For each* $j \in \mathcal{N}_i$ *if* $\Delta^u_{ij} \geq 0$ *then set* $x^c_{ij} = 1$*, otherwise set* $x^c_{ij} = 0$.

2. *If* $\sum_j q_{ij}x^c_{ij} \geq d \sum_j q_{ij}$ *then* $x^*_{ij} = x^c_{ij}$.

3. *If* $\sum_j q_{ij}x^c_{ij} < d \sum_j q_{ij}$ *then use the following procedure:*

   (a) *Sort those indices* $j \in \mathcal{N}_i$*, in descending order according to the quotient* Δ^u_{ij}/q_{ij}*. Let* j_l *denote the corresponding index in the original list of the* l^{th} *element in the sorted list.*

   (b) *Find* l^* *as the minimum* l *satisfying* $\sum_{m=1}^{l} q_{ij_m} \geq d \sum_{j=1}^{N_i} q_{ij}$.

   (c) *Set* $x_{ij_l} = 1$ *for* $l = 1, \ldots, l^*$*;* $x_{ij_l} = 0$ *for* $l = l^* + 1, \ldots, N_i$.

The proposition states that the candidate solution based on the optimal solutions of the N_i subproblems would be optimal for the overall problem for category i, if constraint (11) is satisfied. In the case where constraint (11) is violated by the candidate solution, one needs to set a larger number of variables to one, which is achieved by means of the simple procedure described in the proposition. In the latter case, one deals essentially with a reverse knapsack-type problem, and the proposed procedure is typical for this class of problems.

3.5 Linearization

To compute an optimal solution to the non-convex, nonlinear formulation (6), we develop an efficient linearization scheme equivalent to formulation (6). Recall that

$$Z_i = \sum_{j=1}^{N_i} \left((p_{ij} q_{ij} (1 - s_i) - c) x_{ij} + s_i \left(\sum_{j=1}^{N_i} q_{ij} \right) \frac{\sum_j p_{ij} q_{ij} x_{ij}}{\sum_j q_{ij} x_{ij}} \right).$$

We shall force variable y_i take on the value $\frac{\sum_j p_{ij} q_{ij} x_{ij}}{\sum_j q_{ij} x_{ij}}$ $\forall i \in \mathcal{K}$. If the denominator $\sum_j q_{ij} x_{ij} > 0$ (a condition guaranteed by (9)), the above relation can be forced with the equation:

$$\sum_j p_{ij} q_{ij} x_{ij} = \sum_j q_{ij} y_i x_{ij} \quad i \in \mathcal{K}.$$

Let

$$z_{ij} = y_i x_{ij} \;\; i \in \mathcal{K}, j \in \mathcal{N}_i. \tag{22}$$

Proposition 4 *Let $M_i = max_j p_{ij}$ for all $i \in \mathcal{K}$. Then, for each $i \in \mathcal{K}$ the following model correctly linearizes the original decomposed model (10-11):*

$$max \sum_j (p_{ij} q_{ij} (1 - s_i) - c) x_{ij} + s_i \sum_j q_{ij} y_i \tag{23}$$

$$\textit{subject to:} \qquad \sum_j p_{ij} q_{ij} x_{ij} = \sum_j q_{ij} z_{ij} \qquad j \in \mathcal{N}_i \tag{24}$$

$$z_{ij} \leq y_i \quad j \in \mathcal{N}_i \tag{25}$$

$$z_{ij} \geq y_i - M_i (1 - x_{ij}) \quad j \in \mathcal{N}_i \tag{26}$$

$$z_{ij} \leq M_i x_{ij} \quad j \in \mathcal{N}_i \tag{27}$$

$$\sum_{j=1}^{N_i} q_{ij} x_{ij} \geq d \sum_{j=1}^{N_i} q_{ij} \quad j \in \mathcal{N}_i \tag{28}$$

$$x_{ij} \in \{0, 1\} \quad j \in \mathcal{N}_i. \tag{29}$$

Proof: We are going to use the classical big-M linearizations of equations (22). Indeed, we would like to force the following relationships:

$$z_{ij} = \begin{cases} y_i & \text{if } x_{ij} = 1 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \quad i \in \mathcal{K}, \;\; j \in \mathcal{N}_i, \tag{30}$$

which are equivalent to (22).

For M_i large enough, inequalities (25)-(27) force relationship (30). If $x_{ij} = 1$ then inequalities (25) and (26) force $z_{ij} = y_i$, where inequality (27) becomes a redundant constraint. On the other hand, if $x_{ij} = 0$, then inequality (27) forces $z_{ij} = 0$ where both (25) and (26) become redundant constraints. For these linearizations to work properly M_i should be at least as large as the maximum value each y_i and consequently each z_{ij} can take. An appropriate such value for M_i is found from the following inequality:

$$y_i = \frac{\sum_j p_{ij} q_{ij} x_{ij}}{\sum_j q_{ij} x_{ij}} \leq \max_j p_{ij} \frac{\sum_j q_{ij} x_{ij}}{\sum_j q_{ij} x_{ij}} = M_i.$$

■

To strengthen the linear model (23)-(29), we also added the following valid inequality for each $i \in \mathcal{K}$ to our model

$$\sum_{j=1}^{N_i} p_{ij} q_{ij} x_{ij} \geq y_i d \sum_{j=1}^{N_i} q_{ij},$$

which proved very effective in our numerical experiments.

4 Case Study at Two Local Chains

We developed our model upon the request of a major international FMCG conglomerate which has significant presence in the Turkish market. The company has the largest market share in Turkey in its core operating sectors: Fabric and Home Care products, Health and Beauty Care products, Feminine Care products and Baby Care products. The company wanted us to provide a tool that small and medium size supermarkets in Turkey could use to optimize the assortment on their shelves. They believed that if the supermarkets would eliminate the unprofitable SKUs from their shelves, the visibility of their own products, which are usually the product leaders, would increase and thereby, their sales would be positively affected. Under such a scenario, the provided tool would assist the creation of a win-win situation for both the supermarket and the FMCG conglomerate. They shared their intentions with two local supermarket chains in Ankara, Turkey which in return accepted to collaborate with us. These supermarket chains are considered as part of the "Local Chain" customer channel that constitutes %35 of the FMCG company's total sales volume in the country.

In this case study, we discuss the results of our model on the assortment of shampoo products at two local chains in the Ankara region. Since these two supermarkets were only keeping monthly sales and profitability data, our model was originally shaped around these input requirements. The two chains agreed to disclose the requisite data for the purposes of the project. Although they stated that they could use such a tool for different product classes, they first wanted us to demonstrate the effectiveness of our methodology in a specific product class. We decided to focus on the shampoo products since this product class has the maximum number of SKUs on retailers' shelves while frequent new product launches are still pushing the number higher. Therefore, the choice of shampoo class is definitely one where efficient assortment and shelf usage is to be beneficial. Moreover, since the FMCG company has a significant presence in this class, they agreed with this choice which could help increase the visibility of their market leader products.

The study concentrates on two local chains, that will be referred to henceforth as "Local Chain A" and "Local Chain B", and on three stores of each local chain classified according to their sizes as "small", "medium" and "large". The shampoo SKUs under scrutiny are classified into three categories according to their quality levels and prices, the premium quality in category 1, the middle quality in category 2, and low quality in category 3.

After consulting with retail managers we decided that setting the cost per period of keeping a SKU on the shelf, parameter c in our model, to 1 local currency unit (YTL) is a reasonable choice. We investigate the sensitivity of our results to perturbations in this value in Experiment 3 below.

A preliminary analysis of sales data according to categories, brands and store sizes yielded the following results:

- Although some SKUs are sold in smaller quantities, they bring larger profit,
- Results change according to store size,
- There exist seasonal effects in sales; especially during the summer months noticeable increases in sales of shampoo occur,
- There are promotions in some SKUs affecting the sales figures,
- Prices may fluctuate over months.

The local chain managers reconsider the SKUs to be carried on their shelves twice a year. As a result, we decided to use data corresponding to six calender months each time an instance of the model is solved. But, in order to accommodate the seasonal effects in demand, the data of each six months period is employed to decide on the assortment of the next period corresponding to the same months of the year (using the data with a lag of six months). This means we use the data of the previous period corresponding to the same season for the current assortment. Average prices for SKUs were used in our calculations to eliminate possible price fluctuations that may have occurred during the six month period. Note that the six month period is long enough for the averages to incorporate both promotion and regular price epochs.

It should be noted that the model we propose only deletes SKUs from the current assortment. It does not have a feature to incorporate new SKUs that may become available in time, or old SKUs whose popularity may rise due to new marketing efforts. In order to accommodate the situation, we propose the chains to add the SKUs that look promising to their assortment for a period of six months. After that our methodology would take care of the decision of keeping them on shelves or delisting them.

One should also note that as the number of SKUs carried at the retailer's shelves decreases due to our model, the product pollution would decrease. This would allow the retailer to reorganize its shelves in a more orderly and attractive fashion. Thus, we would actually expect this decrease to give rise to some additional demand from visiting customers who may have been previously overwhelmed by the sheer number of available SKUs and product pollution, and consequently left the shop without buying any shampoo. This model does not take into account this additional demand since it cannot be predicted based on the sales and profitability data available. This only means that under more realistic considerations, we would eliminate even more SKUs and that the estimated profit in the objective function would be greater.

4.1 Calculation of the substitution ratio

We observed that if a desired SKU is out of stock, a fraction of the demand for that particular SKU will be distributed among other shampoos of the same category. This switching behavior is quantified using a parameter s that we refer to as the substitution ratio. According to a marketing survey commissioned upon the request of the FMCG company, a customer who cannot find his/her desired shampoo

a. buys another brand with a probability of 0.16

b. leaves the store without buying shampoo with a probability of 0.10

c. buys the same SKU from another store with a probability of 0.53

d. buys another SKU of the same brand with a probability of 0.15

e. delays shopping with a probability of 0.06

Our main observation here is that the substitution ratio is stable within a product category. That is to say, when a customer comes to buy a relatively cheaper product (third quality in our particular case study) and faces an out-of-stock situation, he/she buys another product in the same category as the one he/she was looking for. However, a customer looking for a first quality shampoo will be less likely to switch to a lower quality shampoo when his/her preferred shampoo is not available. Unfortunately, we have no access to data to estimate different substitution ratios for different quality categories. Therefore, we decided to work with a single aggregate substitution ratio estimate valid for all quality classes. This aggregate estimate was calculated as follows using the previously mentioned marketing survey. We made the assumption that customers will keep using shampoo for hair care in

Store Size	Initial Profit	d	obj	CPU	A_1	B_1	C_1	A_2	B_2	C_2	A_3	B_3	C_3	C
Large	854.5	0.6	875.93	3.27	0.98	0.995	0.995	0.705	0.923	0.936	0.7	0.969	0.967	0.967
		0.8	875.93	0.25	0.98	0.995	0.995	0.705	0.923	0.936	0.7	0.969	0.967	0.967
		0.9	875.93	0.19	0.98	0.995	0.995	0.705	0.923	0.936	0.7	0.969	0.967	0.967
Medium	161.4	0.6	200.92	0.84	0.455	0.727	0.768	0.449	0.689	0.706	0.452	0.802	0.814	0.754
		0.8	199.13	0.76	0.545	0.808	0.838	0.536	0.801	0.796	0.452	0.802	0.814	0.814
		0.9	190.96	0.5	0.682	0.909	0.92	0.681	0.901	0.888	0.645	0.903	0.906	0.903
Small	138.3	0.6	189.27	0.98	0.52	0.749	0.765	0.376	0.602	0.651	0.276	0.639	0.648	0.699
		0.8	180.11	0.34	0.58	0.8	0.811	0.576	0.801	0.827	0.517	0.81	0.818	0.819
		0.9	169.39	0.39	0.72	0.91	0.904	0.718	0.9	0.911	0.69	0.906	0.907	0.907

Table 1: Results for Local Chain A Stores

Store Size	Initial Profit	d	obj	CPU	A_1	B_1	C_1	A_2	B_2	C_2	A_3	B_3	C_3	C
Large	1169.8	0.6	1200.67	0.327	0.941	0.992	0.992	0.742	0.961	0.98	0.528	0.77	0.786	0.975
		0.8	1200.48	0.22	0.941	0.992	0.992	0.742	0.961	0.98	0.583	0.815	0.831	0.978
		0.9	1198.73	0.15	0.941	0.992	0.992	0.742	0.961	0.98	0.694	0.901	0.902	0.981
Medium	578.4	0.6	600.44	0.17	0.978	0.996	1	0.836	0.957	0.994	0.806	0.961	0.970	0.99
		0.8	600.44	0.16	0.978	0.996	1	0.836	0.957	0.994	0.806	0.961	0.970	0.99
		0.9	600.44	0.08	0.978	0.996	1	0.836	0.957	0.994	0.806	0.961	0.970	0.99
Small	58.8	0.6	108.75	0.94	0.395	0.606	0.626	0.352	0.603	0.643	0.441	0.617	0.616	0.63
		0.8	98.28	0.36	0.581	0.801	0.808	0.563	0.801	0.819	0.647	0.802	0.804	0.811
		0.9	88.88	0.19	0.698	0.9	0.904	0.718	0.9	0.916	0.794	0.91	0.911	0.911

Table 2: Results for Local Chain B stores

the long run, and those customers “buying nothing” in the group b. above will come back to the same store with a 0.5 probability. Adding to this figure, the probabilities from a, d and e groups above, we obtain the estimate 0.42 for s used in the rest of this study. We also provide numerical results below for the sensitivity of our findings to the particular value of the substitution ratio.

4.2 Results

We used GAMS/CPLEX 10 system to solve the linear optimization model introduced in Subsection 3.5 to optimality for each of the three stores of Local Chain A and those of Local Chain B. We summarize below our results in three experiments.

Experiment 1: The impact of the minimum conservation ratio. In Tables 1 and 2 below we report the results of our computational experiments for both Local Chain A and Local Chain B, respectively, with the substitution ratio s and cost parameter c fixed, under three different values of the minimum conservation ratio d, namely 0.6, 0.8 and 0.9. The first column under the heading "initial profit" gives the initial total estimated profits using the data made available for this study. Under the column "obj" we report the total estimated profit figure after elimination. The column "cpu" gives the solution time of the linearized model in seconds. The quantities A_i, B_i, C_i, $i = 1, 2, 3$ and C are defined as follows:

$$A_i = \frac{\sum_j x_{ij}}{N_i}, \quad B_i = \frac{\sum_j q_{ij} x_{ij}}{\sum_j q_{ij}}, \quad C_i = \frac{\sum_j p_{ij} q_{ij} x_{ij}}{\sum_j p_{ij} q_{ij}}, \quad \text{and,} \quad C = \frac{\sum_{ij} p_{ij} q_{ij} x_{ij}}{\sum_{ij} p_{ij} q_{ij}}.$$

The quantity A_i represents the ratio of the number of SKUs kept on the shelf after elimination to the total number of SKUs initially on shelf in category i (SKU retained ratio). Similarly, we use B_i to denote the ratio of sales volume from the SKUs kept on the shelf to the total sales volume initially in category i (volume retained ratio). In C_i we record the ratio of the profit due to the SKUs kept on the shelf to the total profit initially from category i (value retained ratio). Finally, C is used to quantify the total profit ratio across all categories (overall value retained ratio).

The reader should note that d imposes a lower bound on the volume retained ratio at each category (B_i). For the Local Chain A Large Store, the value of d does not affect the results. In the Local Chain A Medium and Small stores, we observe –based on A_i values– decreases in the eliminated SKUs in all categories by an approximately equal factor as the minimum conservation ratio increases. In a more stringent elimination effort (lower values of d) larger number of SKUs from lower quality categories are removed compared to the premium category. This result agrees with the intuition that higher quality, higher price SKUs resist elimination better than lower quality, lower price ones since the former have potentially a larger contribution to the total profitability of the store. In the Local Chain B's Large and Medium stores the minimum conservation ratio d has little or no effect on elimination. In cases the model suggests a considerable elimination when it is not tightly constrained by d (when $d = 0.6$), the number of eliminated SKUs significantly decreases as d increases. In the Small store, a more discernible elimination takes place in all categories while it is slightly more pronounced in the middle category.

We observe that the model advocates a sizable elimination in the Small stores, an indication of poor shelf management practices in these stores. The local chains seem to have a tendency to cram too many SKUs even when the space or demand patterns do not justify it. It seems advisable that smaller stores get a handle on profitability increases by keeping a reduced assortment where the reduction should be more severe in the lower quality and lower price categories. In other words, avoiding product pollution seems to be particularly beneficial for smaller stores. This observation is also supported by a more marked profit increase due to elimination in the smaller stores.

It is interesting that the optimal solution of the model suggests that there is a consistent ascendant

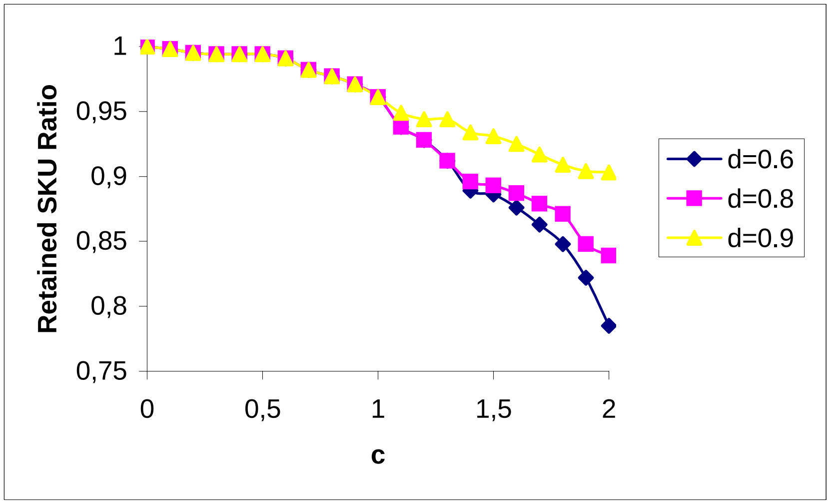

Figure 1: Overall SKU retained ratio versus c

ordering in the SKUs retained, volume retained and value retained at each category. The model tries to keep the SKUs that contribute a larger share of the original profit while performing elimination. In order to achieve this, it has to conserve SKUs contributing more to the sales volume as well –although to a lesser extent–, since sales volume is one of the two determinants of profit along with the price. The SKUs retained trails the previous two measures, since in order to maximize profit one has to keep a certain number of SKUs, but eliminate SKUs with limited contribution to the profit.

Experiment 2: The impact of c. In Figures 1 to 2 we illustrate the impact of the cost parameter c on the elimination of SKUs. We select Local Chain A Large store as our test store. In Figure 1 we report the overall SKU retained ratios ($\frac{\sum_{ij} x_{ij}}{\sum_i N_i}$) achieved by the results of our model at different values of minimum conservation ratios and increasing c from 0 to 2 in steps of 0.1. As expected, the larger values of c, the cost of keeping SKUs on the shelf, lead to increased elimination patterns. In Figure 2 we report SKU retained ratio in all three categories for the value of d fixed to 0.8. Interestingly, the middle category SKUs seem to be more susceptible to sharper elimination as c increases than the other categories. Apparently, the model sheds more SKUs from the middle category than from the extreme categories as the cost of keeping SKUs on the shelf goes up. Similar observations are made when the value of d is 0.6 or 0.9.

Experiment 3: The impact of s. While we adopted the value of $s = 0.42$ in our previous experiments, we tested the sensitivity of the results to different values of s ranging from 0.35 to 0.5 using the Local Chain A Large store for illustration. Figure 3 summarizes the results of this

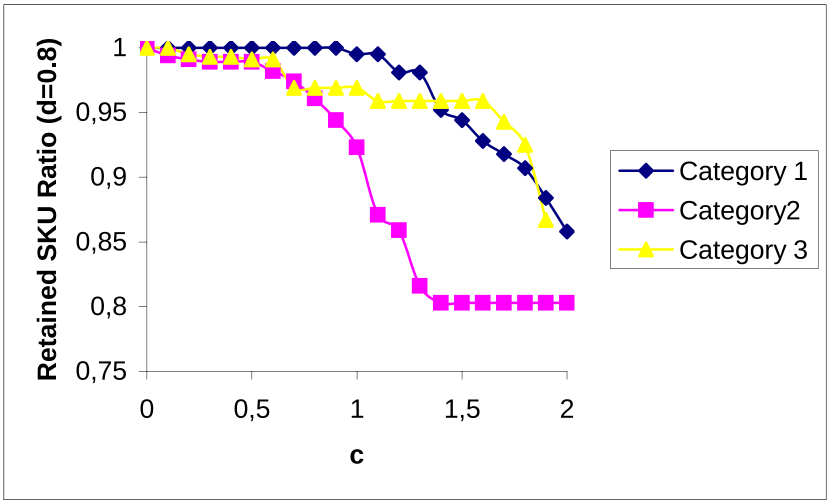

Figure 2: SKU retained ratio for all three categories versus c at $d = 0.8$

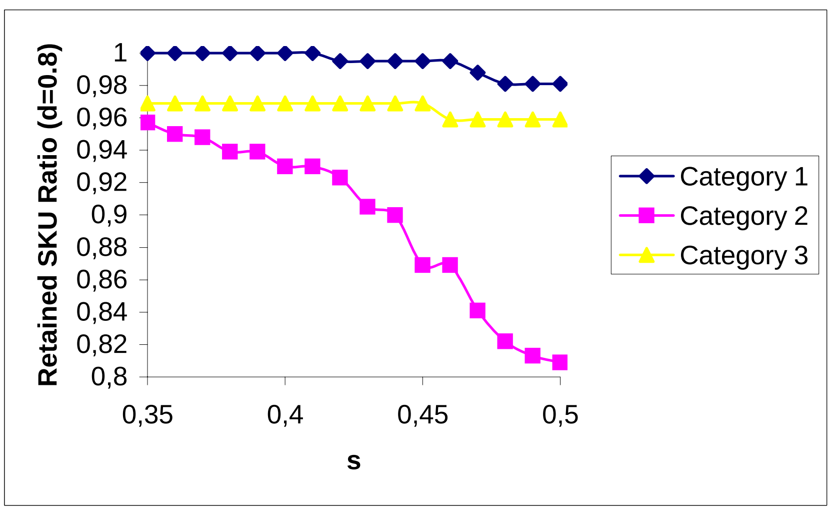

Figure 3: Retained SKU ratio versus s at $d = 0.8$

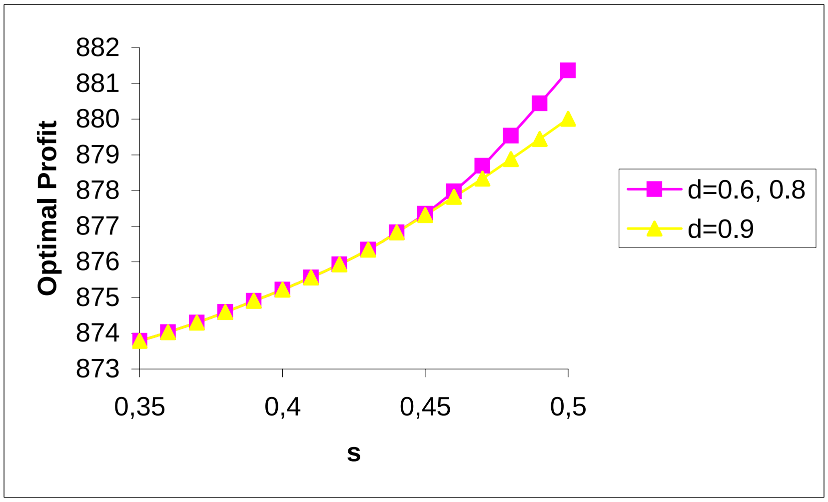

Figure 4: Optimal objective function value versus s

experiment at $d = 0.8$. An observation similar to that made in Experiment 2 prevails here. The middle category SKUs are clearly more sensitive to perturbations in the substitution ratio s than the extreme categories. The changing values of s do not seem to affect much the high quality and low quality categories. This reflects the existence of an abundance of SKUs exhibiting a wide spectrum of profitability profile, which responds to increases in the substitution ratio by gradually enlarging the set of delisted SKUs. It is to be expected that the low quality category does not respond to increased substitution ratio, since the range of prices –and thereby profitability– is rather limited in this category and that does not create an opportunity for capturing profit by substitution. Similar observations are made when the value of d is 0.6 or 0.9.

The impact of the substitution ratio on the optimal objective function value for different values of d in Figure 4 indicates a convex increasing curve. Higher substitution ratios increase total profit faster than the corresponding increase in substitution. This impact is less pronounced at higher minimum conservation ratios, i.e., less severe elimination, as expected.

5 Conclusion

In this paper we developed a practical optimization model that improves the product mix on the shelves of a retailer, and thus reduces product pollution. We applied the proposed model to two local supermarket chains using proprietary data disclosed to us by the chain managers. This project was undertaken upon the request of an FMCG conglomerate that supplies these –as well as others– retail channels. Their motivation to initiate such an endeavor was to persuade the local supermarket chains that a limited assortment would benefit their profitability, while simultaneously solving the visibility problems faced by the market leader products of the FMCG company. To continuously guide the chain manager on the correct assortment, they were asking for a simple and practical tool. We developed such a tool based on the model presented in this paper. To demonstrate the usefulness of our tool, we decided to first focus on the highly problematic shampoo class. Our results that were summarized in Section 4 revealed that a judicious elimination based on our model has the potential for a significant increase in profitability. The FMCG company as well as the studied supermarket management are in the process of incorporating this decision support tool into their operations.

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